We want to compute:

$$E\left[\max_i v_i \epsilon_i\right].$$

First, let random variable X be $\max_{i \in \{1,\cdots,n\}} v_i \epsilon_i$.

And let G be the distribution function of X. We derive G.

$$\begin{aligned}
& G(x) \\
&= P(X \leq x) \\
&= P\left(\max_i v_i \epsilon_i \leq x\right) \\
&= P\left(v_1 \epsilon_1 \leq x \text{ and } v_2 \epsilon_2 \leq x \text{ and } \cdots \text{ and } v_n \epsilon_n \leq x\right) \\
&= P(v_1 \epsilon_1 \leq x) \cdot P(v_2 \epsilon_2 \leq x) \cdot \cdots \cdot P(v_n \epsilon_n \leq x)
\end{aligned}$$

$$= P\left(\epsilon_1 \leq \frac{x}{v_1}\right) \cdot P\left(\epsilon_2 \leq \frac{x}{v_2}\right) \cdots P\left(\epsilon_n \leq \frac{x}{v_n}\right).$$

$$= F\left(\frac{x}{v_1}\right) \cdot F\left(\frac{x}{v_2}\right) \cdots F\left(\frac{x}{v_n}\right).$$

$$= e^{-\left(\frac{x}{v_1}\right)^{-\theta}} \cdot e^{-\left(\frac{x}{v_2}\right)^{-\theta}} \cdots e^{-\left(\frac{x}{v_n}\right)^{-\theta}}$$

$$= e^{-\sum_{i=1}^{n}\left(\frac{x}{v_i}\right)^{-\theta}}$$

$$= e^{-x^{-\theta}\sum_{i=1}^{n} v_i^{\theta}}.$$

Let $\alpha = \sum_{i=1}^{n} v_i^{\theta}$.

Then,

$$G(x) = e^{-x^{-\theta}\cdot\alpha}.$$

The density function of X is

$$g(x) = \theta\alpha x^{-\theta-1} e^{-x^{-\theta}\cdot\alpha}.$$

Then what we wanted is

$E[X]$

$$= \int_0^\infty x \cdot g(x)\, dx$$

$$= \int_0^\infty x \cdot \theta \alpha x^{-\theta-1} e^{-x^{-\theta}\alpha}\, dx.$$

$$= \int_0^\infty \theta \alpha x^{-\theta} e^{-x^{-\theta}\alpha}\, dx.$$

Let $t = x^{-\theta}\alpha$. $\quad x^{\theta} = \frac{\alpha}{t}$. $\quad x = \left(\frac{\alpha}{t}\right)^{\frac{1}{\theta}}$.

x	$0 \to \infty$
t	$\infty \to 0$.

$$\frac{dt}{dx} = -\theta x^{-\theta-1}\alpha$$

$$= -\theta t x^{-1}$$

$$= -\theta t \left(\frac{\alpha}{t}\right)^{-\frac{1}{\theta}}.$$

$$= -\theta t^{1+\frac{1}{\theta}} \alpha^{-\frac{1}{\theta}}$$

$$dx = -\frac{dt}{\theta t^{1+\frac{1}{\theta}} \alpha^{-\frac{1}{\theta}}}.$$

Therefore, the integration is

$$\int_{\infty}^{0} \theta t e^{-t} \cdot \left(- \frac{dt}{\theta t^{1+\frac{1}{\theta}} \alpha^{-\frac{1}{\theta}}} \right).$$

$$= \int_{0}^{\infty} t^{-\frac{1}{\theta}} \alpha^{\frac{1}{\theta}} e^{-t} dt$$

$$= \alpha^{\frac{1}{\theta}} \int_{0}^{\infty} t^{-\frac{1}{\theta}} e^{-t} dt$$

$$= \alpha^{\frac{1}{\theta}} \int_{0}^{\infty} t^{(1-\frac{1}{\theta})-1} e^{-t} dt.$$

$$= \alpha^{\frac{1}{\theta}} \cdot \Gamma\left(1 - \frac{1}{\theta}\right),$$

where the last equality follows from the definition of the gamma function

$$\Gamma(x) = \int_{0}^{\infty} t^{x-1} e^{-t} dt.$$

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