# Why Not the General Fréchet?

Motoaki Takahashi

University of Osaka

May 21, 2026

# Two distributions

- In the main slides, we have used a standard Fréchet distribution

$$F(x) = e^{-x^{-\theta}}. \tag{1}$$

- But, if we look at Wikipedia, there is a more generalized version

$$F(x) = e^{-\left(\frac{x-m}{s}\right)^{-\theta}}, \tag{2}$$

  - $\theta$: shape parameter,
  - $s$: scale parameter,
  - $m$: location parameter.
- Why didn't we use the latter, more general one?

# Writing a Fréchet as a linear transformation of a standard Fréchet

- Let $Z$ follow the standard Fréchet. Then a (general) Fréchet, $\epsilon$, is expressded as

$$\epsilon = m + sZ. \tag{3}$$

- If $m \neq 0$, the share and welfare formulae would be complicated and ugly.
  - $v_i \epsilon = v_i m + v_i sZ$. Therefore, this form is no longer multiplicative.
  - And the share and welfare formulae would not take the same form as CES does.

## $m = 0$ but $s \neq 1$

- Now we assume $m = 0$ but $s \neq 1$.
- Then the share equation is the same as the case of $s = 1$

$$P[v_i \epsilon_i > v_j \epsilon_j \text{ for any } j \neq i] = \frac{v_i^\theta}{\sum_{j=1}^n v_j^\theta}. \tag{4}$$

- The welfare formula is proportinal to the one for $s = 1$

$$E\left[\max_{i \in \{1, \cdots, n\}} v_i \epsilon_i\right] = s \cdot \Gamma\left(1 - \frac{1}{\theta}\right) \cdot \left(\sum_{i=1}^n v_i^\theta\right)^{1/\theta}. \tag{5}$$

- $s$ does not change any relevant equilibrium objects.
  - The level of welfare is not an object of interest.
  - The ratio of welfare between two equilibria is of interest.
  - And based on the share formula, there is not way to identify $s$ from data.
- Therefore, we normalize $s$ by setting $s = 1$.