Housing Constraints and Spatial Misallocation
Hsieh and Moretti (2019)

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Introduction (1)

- Now we are equipped with most of mathematical techniques to understand spatial models.
- From today, we study quantitative spatial models.
- Quantitative spatial models have been actively studied in the last 10+ years.
  - The few earliest publications are Desmet and Rossi-Hansberg (2014) and Allen and Arkolakis (2014).

Introduction (2)

- Today we study Hsieh and Moretti (2019) "Housing Constraints and Spatial Misallocation" and Greaney (2025)'s criticism.
- Housing supply elasticities are different across U.S. cities.
  - Usually, supply elasticities are defined as how much supply increases if the price increases by one percent.
  - A natural counterpart in the housing market is how much housing supply increases if the rent or housing price increases by one percent.
  - This is a solid concept. Nothing is wrong.
  - But, somehow, urban economists often consider how much housing supply increases if the population increases by one percent.
  - Or, how much housing rents increase if the population increases by one percent (the inverse elasticity).
  - The model of Hsieh and Moretti (2019) uses the last concept (rents against population).

Introduction (3)

- How rents respond to populations is different across U.S. cities.
- Suppose the housing supply is perfectly elastic.
- Then, even if people move in a location, many houses are built immediately, and rents won't go up.
- Suppose the housing supply is perfectly inelastic.
- Then, if people move in a location, no house is additionally built, and rents will skyrocket.
- Then what determines housing supply elasticities?
- Although this question is not the focus of this paper, Saiz (2010) points out two primary drivers.
  1. Topography
     - It is difficult to build houses on a steep hill.
  2. Regulation
     - One needs to submit a 100-page document to build an apartment complex.
     - Rules require that houses need at least 10 square meters buffer areas, three windows, one emergency door, etc.

Questions in this paper

- Housing regulations or constraints are known to be strict in big, productive U.S. cities: New York and the San Francisco Bay Area.
- These cities are driving productivity growth in the U.S.
- What if housing constraints are less strict in these cities?
- Labor (people) should be more allocated to these productive cities.
- And U.S. aggregate productivity should be higher.
- By how much?

Model

Production function

- There are N cities. Cities are indexed by $i = 1, \cdots, N$.
- There are L units of individuals (not specified in the original paper, though).
- City $i = 1, \cdots, n$ produces freely tradeable, homogeneous goods with the production function

$$Y_i = A_i L_i^{\alpha} K_i^{\eta} T_i^{1-\alpha-\eta},$$

  where
  - A_i is the productivity in i,
  - L_i is the employment in i,
  - K_i is the capital in i,
  - T_i is the land available for business use in i.
- All cities produce the same product.
- The Cobb-Douglas parameters (α and η) are the same across cities.
  - The labor share, capital share, and land share in value-added are the same across cities.

Assumption on factor markets and labor market clearing

- In each city i, land supply T_i is fixed.
- Capital markets are completely integrated across all cities.
- The interest rate R is exogenously determined in world capital markets.
- Equating the marginal product of labor and capital to the local nominal wage W_i and the (global) cost of capital R, the local labor demand is

$$L_i = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}} \cdot \frac{A_i}{W_i^{1-\eta}} \right)^{\frac{1}{1-\alpha-\eta}} \cdot T_i. \tag{1}$$

  - It is increasing in A_i and T_i and decreasing in W_i.
- We call the composite $A_i^{\frac{1}{1-\alpha-\eta}} T_i$ as "local TFP."

Wage equation

- Rewriting (1), we have

$$W_i = \mathcal{A}_i L_i^{-\frac{1-\alpha-\eta}{1-\eta}}, \tag{2}$$

where

$$\mathcal{A}_i = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}} A_i T_i^{1-\alpha-\eta} \right)^{\frac{1}{1-\eta}}.$$

- This is the way Greaney (2025) writes the condition.

Workers' location choices

- The indirect utility of worker j in city i is

$$V_{ji} = \epsilon_{ji} \cdot \frac{W_i Z_i}{P_i^{\beta}},$$

  where

  - ϵ_{ji} is individual j's idiosyncratic preference shock for city i,
  - P_i is the housing price in i,
  - β is the expenditure share on housing (a parameter),
  - Z_i is the value of local amenities in i.

- Behind this, the authors assume the Cobb-Douglas utility function of freely tradeable, homogeneous goods (the numeraire) and locally suppled housing.

Fréchet

- Assume that ϵ_{ji} are independently distributed and drawn from

$$F_g(\epsilon_1, \cdots, \epsilon_N) = \exp\left(-\sum_{i=i}^{N} \epsilon_i^{-\theta}\right).$$

- Then the population share of city i is

$$\frac{L_i}{L} = \frac{(W_i Z_i P_i^{-\beta})^\theta}{\sum_j (W_j Z_j P_j^{-\beta})^\theta}. \tag{3}$$

- This is exactly the same formula as we saw in "Discrete choices and extreme value distributions."

Inverse local labor supply

- Normalize the total population to one.
  - Greaney (2025) criticizes this point.
- The ex ante expected utility is

$$V = \left(\sum_j (W_j Z_j P_j^{-\beta})^\theta \right)^{1/\theta}$$

ignoring a constant from the gamma function.
- Then (3) is now

$$L_i = \frac{(W_i Z_i P_i^{-\beta})^\theta}{V^\theta}.$$

- Solving this for W_i, the (inverse) local labor supply to a city is

$$W_i = V \cdot \frac{P_i^\beta L_i^{1/\theta}}{Z_i}. \tag{4}$$

Housing market (1)

- This is a controversial part.
- Hsieh and Moretti assume that the local housing price is given by

$$P_i = \bar{P}_i L_i^{\gamma_i}, \tag{5}$$

  where
  - γ_i is the (inverse) elasticity of housing supply with respect to the number of workers in the city,
  - $\bar{P}_i$ denotes the part of the local housing price that does not vary with employment.
- P_i is endogenous and a function of L_i (which is also endogenous); $\bar{P}_i$ is a parameter.
- γ_i are different across cities.
- Cities with a limited amount of land or stringent land use regulations have a lower elasticity of housing supply (large γ_i).
- Cities with abundant land or permissive land use regulations have higher elasticity (small γ_i).

Housing market (2)

- But, Greaney (2025) asserts that the specification (5) leads to unit dependence.
- With Hsieh and Moretti's original specification (5), key equilibrium outcomes including the effects of changes in γ_i on welfare and aggregate output depend on the total population L.
- The original paper quietly sets the total population to one and regards it as an innocuous assumption.
- (Overall, I think Hsieh and Moretti (2019) propose a nice, simple model for an interesting question, though.)

Housing market (3)

- To resolve this unit dependence, Greaney proposes the following house price specification

$$P_i = \bar{P}_i \left(\frac{l_i g}{l_{i,1970}} \right)^{\gamma_i}, \tag{6}$$

  where
  - l_i is city i's population share (as an endogenous object in the model),
  - $g = L/L_{1970}$ is total population growth since 1970 (exogenous),
  - $l_{i,1970}$ is city i's population share as of 1970 in data (exogenous).
- The "main" data year is 2009, so g is the population growth from 1970 to 2009.
- According to Greaney (2025), this specification comes from Saiz (2010).

Equilibrium conditions

- An equilibrium is $(W_i, P_i, L_i)_{i=1}^N$ such that (2), (5), (3):

$$W_i = \mathcal{A}_i L_i^{-\frac{1-\alpha-\eta}{1-\eta}},$$
$$P_i = \bar{P}_i L_i^{\gamma_i},$$
$$\frac{L_i}{L} = \frac{(W_i Z_i P_i^{-\beta})^\theta}{\sum_j (W_j Z_j P_j^{-\beta})^\theta}.$$

- This is the way Greaney summarizes the Hsieh-Moretti model.

Equilibrium conditions of Greaney's version

- Greaney proposes a housing price equation slightly different from the original one.
- Therefore, his version of equilibrium conditions are (2), (6), (3):

$$
\begin{aligned}
W_i &= \mathcal{A}_i L_i^{-\frac{1-\alpha-\eta}{1-\eta}}, \\
P_i &= \bar{P}_i \left(\frac{(L_i/L)g}{l_{i,1970}} \right)^{\gamma_i}, \\
\frac{L_i}{L} &= \frac{(W_i Z_i P_i^{-\beta})^{\theta}}{\sum_j (W_j Z_j P_j^{-\beta})^{\theta}}.
\end{aligned}
$$

Discussions on equilibrium

- Either way, we have $3N$ endogenous variables with $3N$ equations.
- Generally, this does not guarantee the existence of an equilibrium.
- But, in this case, wages are a decreasing function of local populations.
- And housing prices are an increasing function of local populations.
- Therefore, there are only congestion forces in this model.
  - There is no agglomeration force.
- So the unique equilibrium exists.
  - Of course this is not a mathematical proof.
  - We may see a formal proof of existence and uniqueness of a spatial model later.
- Numerically, iterating three equations yields an equilibrium.

Local employment and aggregate output

- Now assume Hsieh and Moretti's version of housing prices (5).
- Then equilibrium employment in a city is

$$L_i = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}V^{1-\eta}} \cdot A_i T_i^{1-\alpha-\eta} \cdot \left(\frac{Z_i}{\bar{P}_i^{\beta}} \right)^{1-\eta} \right)^{\frac{1}{1-\alpha-\eta+\beta(\gamma_i+1/\theta)(1-\eta)}} .$$

- Aggregate output is

$$Y = \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}} \left[\sum_i \left(A_i \cdot \left[\frac{\bar{Q}}{Q_i} \right]^{1-\eta} \cdot T_i^{1-\alpha-\eta} \right)^{\frac{1}{(1-\eta)(1+1/\theta)-\alpha}} \right]^{\frac{(1-\eta)(1+1/\theta)-\alpha}{1-\eta}} ,$$

where

$$Q_i = \frac{P_i^{\beta}}{Z_i}$$

and

$$\bar{Q} = \sum_i L_i^{1+1/\theta} \cdot Q_i .$$

Welfare

- Average welfare is

$$V = \alpha \cdot \frac{Y}{\bar{Q}}.$$

Unit dependence (1)

Greaney

- Why does the original version by Hsieh and Moretti depend on population units, whereas Greaney's version does not?
- γ_i play an important role.
- First we look at Greaney's version and then Hsieh and Moretti's original version.
- Greaney's version is expressed as

$$
\begin{aligned}
W_i &= \mathcal{A}_i (l_i L)^{-\delta}, \\
P_i &= \check{P}_i l_i^{\gamma_i}, \\
l_i &= \frac{(W_i Z_i P_i^{-\beta})^{\theta}}{\sum_j (W_j Z_j P_j^{-\beta})^{\theta}},
\end{aligned}
$$

where $\delta = \frac{1-\alpha-\eta}{1-\eta}$, $\check{P}_i = \bar{P}_i \left(\frac{g}{l_{i,1970}} \right)^{\gamma_i}$, and we recall that $l_i = L_i / L$.

Unit dependence (2)

Greaney

- These three (actually $3N$) equations are condensed as

$$l_i = \frac{[\mathcal{A}_i(l_i L)^{-\delta} Z_i(\check{P}_i l_i^{\gamma_i})^{-\beta}]^\theta}{\sum_j [\mathcal{A}_j(l_j L)^{-\delta} Z_j(\check{P}_j l_j^{\gamma_j})^{-\beta}]^\theta}.$$

- Suppose that total population L changes to aL ($a > 0$).
- Then population share l_i^{new} for such new total population is

$$\begin{aligned} l_i^{\text{new}} &= \frac{[\mathcal{A}_i(l_i aL)^{-\delta} Z_i(\check{P}_i l_i^{\gamma_i})^{-\beta}]^\theta}{\sum_j [\mathcal{A}_j(l_j aL)^{-\delta} Z_j(\check{P}_j l_j^{\gamma_j})^{-\beta}]^\theta} \\ &= \frac{a^{-\delta\theta}[\mathcal{A}_i(l_i L)^{-\delta} Z_i(\check{P}_i l_i^{\gamma_i})^{-\beta}]^\theta}{a^{-\delta\theta}\sum_j [\mathcal{A}_j(l_j L)^{-\delta} Z_j(\check{P}_j l_j^{\gamma_j})^{-\beta}]^\theta} \\ &= l_i. \end{aligned}$$

- Therefore, population units do not affect equilibrium outcomes.

Unit dependence (3)

Hsieh and Moretti

- Now we turn to Hsieh and Moretti's original model.
- The system of equilibrium conditions is

$$W_i = \mathcal{A}_i(l_i L)^{-\delta},$$
$$P_i = \bar{P}_i(l_i L)^{\gamma_i},$$
$$l_i = \frac{(W_i Z_i P_i^{-\beta})^\theta}{\sum_j (W_j Z_j P_j^{-\beta})^\theta},$$

where we recall that $\delta = \frac{1-\alpha-\eta}{1-\eta}$ and that $l_i = L_i/L$.

- These three equations are condensed as

$$l_i = \frac{[\mathcal{A}_i(l_i L)^{-\delta} Z_i \{\bar{P}_i(l_i L)^{\gamma_i}\}^{-\beta}]^\theta}{\sum_j [\mathcal{A}_j(l_j L)^{-\delta} Z_j \{\bar{P}_j(l_j L)^{\gamma_j}\}^{-\beta}]^\theta}.$$

Unit dependence (4)

Hsieh and Moretti

- Suppose that total population L changes to aL $(a > 0)$.
- Then the new population share is

$$\begin{aligned} l_i^{\text{new}} &= \frac{[\mathcal{A}_i(l_i aL)^{-\delta} Z_i\{\bar{P}_i(l_i aL)^{\gamma_i}\}^{-\beta}]^\theta}{\sum_j [\mathcal{A}_j(l_j aL)^{-\delta} Z_j\{\bar{P}_j(l_j aL)^{\gamma_j}\}^{-\beta}]^\theta} \\ &= \frac{a^{-(\delta\theta+\gamma_i\beta\theta)}[\mathcal{A}_i(l_i L)^{-\delta} Z_i\{\bar{P}_i(l_i L)^{\gamma_i}\}^{-\beta}]^\theta}{\sum_j a^{-(\delta\theta+\gamma_j\beta\theta)}[\mathcal{A}_j(l_j L)^{-\delta} Z_j\{\bar{P}_j(l_j L)^{\gamma_j}\}^{-\beta}]^\theta} \\ &\neq l_i. \end{aligned}$$

- Therefore, in Hsieh and Moretti's original model, population units affect the spatial distribution of populations.
- Notice that if the housing supply elasticities are all equal across cities, $\gamma_i = \gamma$ for some $\gamma > 0$, $l_i^{\text{new}} = l_i$ holds.

Data

Data

- The main data are the 1964, 1965, 2008, and 2009 County Business Patterns (CBP).
  - Check the U.S. government's official webpage and Fabian Eckert's webpage.
- The authors supplement this data with
  - the 1960 and 1970 Census of Population,
  - the 2008 and 2009 American Community Survey (ACS),
  - the 1964 and 2009 Current Population Survey (CPS).
- The Census, ACS, and CPS are all available on IPUMS.
- Data on employment and average wages are available at the county and country-industry-level from the CBP.
- The authors aggregate them to Metropolitan Statistical Area (MSA) and MSA-industry level.

Data: toward residual wages (1)

- A strength of the CBP data is that they date back to 1964.
- A limitation of them is that they do not provide worker-level information.
- The authors augment the CBP data with MSA-level information on worker characteristics from the Census of Population, the ACS, and the CPS:
  - three levels of education attainment
    - high school dropout,
    - high school,
    - college,
  - race,
  - gender,
  - age,
  - union status.

Data: toward residual wages (2)

- Basically Hsieh and Moretti want to eliminate composition effects in wages.
- There are many possible reasons why the wage in a city is high.
- Productivity is inherently high in the city; highly educated people just gather in the city.
- There is a long argument about whether wage gaps across cities are driven by sorting.

Data: toward residual wages (3)

- The authors first regress individual wages on worker characteristics in the nationwide data of the 1964 and 2009 CPS.
- Then compute $W - X'b$, where
  - W is a vector of the average wage in the MSA,
  - X is the vector of average worker characteristics in the MSA,
  - b is a vector of coefficients on worker characteristics estimated from individual-level regressions in nationwide samples.

Data: housing costs and housing supply elasticities

- Data on housing costs are measured as median annual rent from the 1960, 1970 US Census of Population and the 2008 and 2009 American Community Survey.
- Housing supply elasticities γ_i are from Saiz (2010).
  - Of course, γ_i are parameters in the model.
  - But, when Saiz (2010) estimates γ_i, they are a function of
    - land availability,
    - land use regulations.

Parameters

- Using National Economics Accounts of Bureau of Economic Analysis, Karabarbounis and Neiman (2013), Piketty and Goldhammer (2014), Hsieh and Moretti set $\alpha = 0.65$ and $\eta = 0.25$.
- Then we can use (1) to calibrate local TFP:

$$A_i^{\frac{1}{1-\alpha-\eta}} \cdot T_i \propto L_i \cdot W_i^{\frac{1-\eta}{1-\alpha-\eta}}.$$

- The authors set $\beta = 0.32$ following Albouy (2008).
  - Some papers (notably Stephen Redding's papers) set $\beta = 0.25$ quoting Davis and Ortalo-Magne (2011).
- They set $1/\theta = 0.3$ following Hornbeck and Moretti (2018).
  - This parameter is too important. We will come back to this parameter later.
- Solving (4) for Z_i, we have

$$Z_i = V \cdot \frac{P_i^{\beta} L_i^{1/\theta}}{W_i}.$$

See Table 1 (p. 13 in the original paper) and Figures 1 to 5 (pages 14 to 18).

Caveats on counterfactuals

- Greaney (2025) points out that most of counterfactual numbers in Hsieh and Moretti (2019) depend on population units.
- So, here I just show Greaney (2025)'s recalculation of Hsieh and Moretti (2019)'s main counterfactual scenario.

Main counterfactual scenario

- The Wharton Index is famous for various housing market measures.
- One of them is stringency on land use regulations.
- The restrictions on land use in New York, San Francisco, and San Jose are among the tightest in the country.
- The elasticity of housing supply in San Francisco is at the ninety-ninth percentile and New York and San Jose at the ninety-sixth percentile.
  - I didn't perfectly understand the corresponding sentence in the original paper, though.
- In the main counterfactual scenario, Hsieh and Moretti shift land use regulations in New York, San Jose, and San Francisco to the level of the median city.
- Other things fixed, they inject new, lower housing supply elasticities in these three big cities to the model.

See Table 2 in page 7, Greaney (2025).

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