

The Determinants and Welfare Implications of  
US Workers' Diverging Location Choices by Skill:  
1980–2000  
Diamond (2016)

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# Introduction

- ▶ From 1980 to 2000, the college/high school graduate wage gap rose in the U.S.
- ▶ In the meantime, college graduates became more concentrated in high wage, high rent cities.
- ▶ This paper estimates a spatial equilibrium model.
- ▶ So that it determines causes and welfare consequences of this increased skill sorting.

## Preview:

- ▶ Local labor demand changes caused the increased skill sorting.
- ▶ But it was further fueled by endogenous increases in amenities within higher skill cities.

## Technical contribution

- ▶ This paper endogenizes amenities as a function of skill composition in cities.
- ▶ "Skills are amenities."

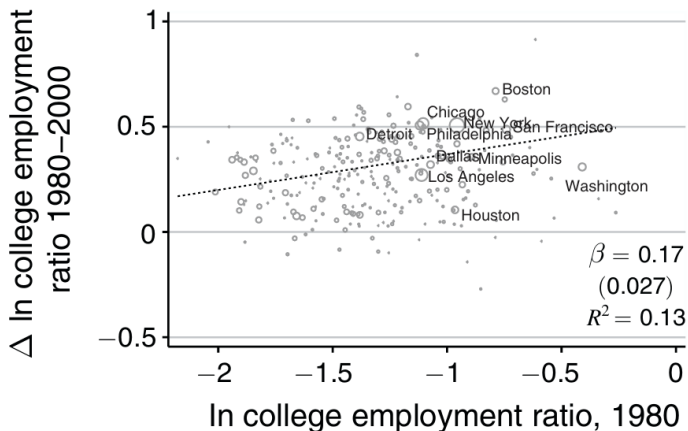
## "Skilled"

- ▶ The author restricts her sample to 25-55-year-olds who work at least 35 hours per week and 48 weeks per year.
- ▶ College (high skill) workers are those who spent 4 or more years in college.
- ▶ All other full-time workers are noncollege (low skill) workers.

# Empirical regularities (1)

College workers are getting more concentrated

Panel A

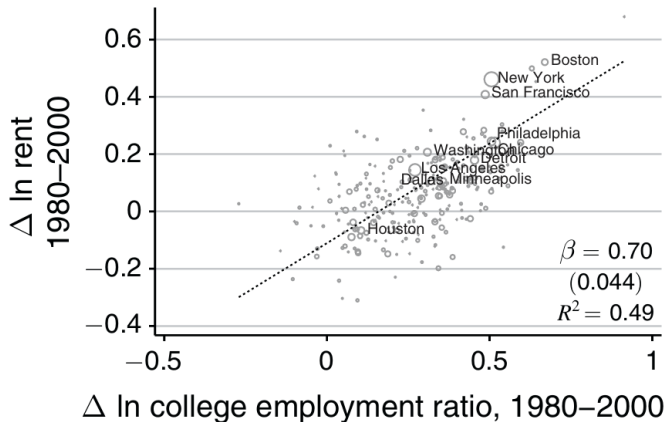


Weighted by 1980 population. MSAs+the non-MSA area in each state.

# Empirical regularities (2)

Rents increase more in cities where college workers get concentrated

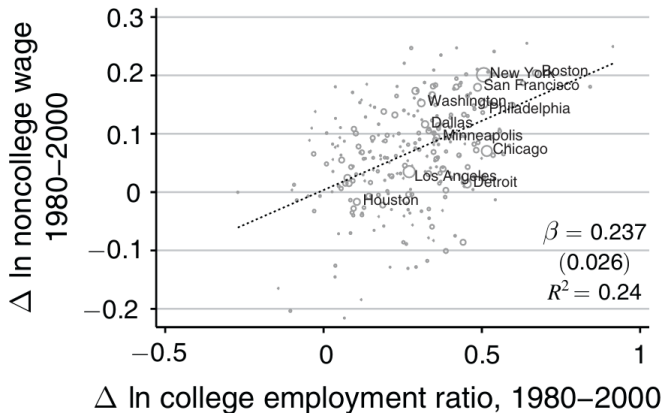
Panel B



# Empirical regularities (3)

Noncollege wages increase more in cities where college workers get concentrated

Panel C



## Empirical regularities (4)

College wages increase more in cities where college workers get concentrated

Panel D



Compare the scales of the vertical axes across Panels B, C, and D.

## Summary of the four graphs

- ▶ Initially skill-intensive cities became more skill-intensive.
- ▶ Rents increased in skill intensive cities.
  - ▶ 0.7 percent for one percent increase in the college employment ratio.
- ▶ Noncollege wages increased more in skill intensive cities, but less so than rents did.
  - ▶ 0.24 percent for one percent increase in the college employment ratio.
- ▶ College wages increased in skill more in skill intensive cities, but less so than rents did.
  - ▶ 0.3 percent for one percent increase in the college employment ratio.
- ▶ College workers are getting concentrated in high wage, high rent cities, where their real wages are not necessarily becoming higher.
- ▶ Noncollege workers are initially and increasingly concentrated in low wage, low rent locations.

## But college workers can voluntarily choose locations

- ▶ Then are college workers getting worse off in expensive cities?
- ▶ But they are free to move out.
- ▶ Instead, more college workers live in such expensive cities.
- ▶ Something is missing in the previous slide: amenities.
- ▶ College workers may be attracted to non-pecuniary amenities in urban areas.

# Correlation between changes in skill intensity and changes in amenity features

TABLE 3—MSA COLLEGE RATIO CHANGES ON AMENITY CHANGES, 1980–2000

*Panel A. Retail amenities*

	Apparel stores per 1,000 residents	Eating and drinking places per 1,000 residents	Movie theaters per 1,000 residents
$\Delta$ College emp. ratio	0.477*** [0.0928]	0.182*** [0.0539]	0.230 [0.166]

*Panel B. Transportation amenities*

	Bus routes per capita	Public transit index	Avg. daily traffic: interstates	Avg. daily traffic: major roads
$\Delta$ College emp. ratio	1.045*** [0.376]	0.0161 [0.338]	-0.169* [0.0979]	-0.0513 [0.0704]

	Property crimes per 1,000 residents	Violent crimes per 1,000 residents	Gov. spending on parks per capita	EPA air quality index
$\Delta$ College emp. ratio	-0.231* [0.122]	0.115 [0.155]	0.263 [0.172]	-0.539*** [0.171]

	Gov. K-12 spend- ing per student	Student-teacher ratio	Patents per capita	Employment rate
$\Delta$ College emp. ratio	0.129** [0.0639]	0.00423 [0.0631]	0.104 [0.234]	0.0105 [0.00787]

Higher values of the EPA air quality index indicate more pollution.

## Amenities increased in cities where college employment ratios increased

- ▶ Changes in the numbers of apparel stores, restaurants, bars, and bus routes are positively correlated with changes in college employment ratios.
- ▶ Changes in interstate traffics, property crimes, and air pollution are negatively correlated with changes in college employment ratios.
- ▶ Changes in government spending on schools per pupil are positively correlated with changes in college employment ratios.
- ▶ So a possible story is as follows.
- ▶ So cities attracting college workers are getting higher amenities.
- ▶ These amenities further attract college workers.
- ▶ And these amenities did not directly show up in the graphs on wages/rents.

Model

## Labor demand

- ▶ Each city, indexed by  $j$ , has many homogeneous firms, indexed by  $d$ , in year  $t$ .
  - ▶ But the model is static.
- ▶ Firms produce a homogeneous tradable good using high skill labor ( $H_{djt}$ ), low skill labor ( $L_{djt}$ ), and capital ( $K_{djt}$ ) according to the production function

$$Y_{djt} = N_{djt}^{\alpha} K_{djt}^{1-\alpha},$$

$$N_{djt} = (\theta_{jt}^L L_{djt}^{\rho} + \theta_{jt}^H H_{djt}^{\rho})^{\frac{1}{\rho}},$$

$$\theta_{jt}^L = f_L(H_{jt}, L_{jt}) \exp(\epsilon_{jt}^L),$$

$$\theta_{jt}^H = f_H(H_{jt}, L_{jt}) \exp(\epsilon_{jt}^H).$$

- ▶  $f_L$  and  $f_H$  represent that skill-specific productivity can depend on high and low skill employment.

## Wages and capital rents: FOC

- ▶ Capital is freely mobile across cities; capital rents are equalized across cities.
- ▶ Firms are competitive; factor prices are equalized to marginal products

$$W_{jt}^H = \alpha N_{djt}^{\alpha-\rho} K_{djt}^{1-\alpha} H_{djt}^{\rho-1} f_H(H_{jt}, L_{jt}) \exp(\epsilon_{jt}^H),$$

$$W_{jt}^L = \alpha N_{djt}^{\alpha-\rho} K_{djt}^{1-\alpha} L_{djt}^{\rho-1} f_L(H_{jt}, L_{jt}) \exp(\epsilon_{jt}^L),$$

$$\kappa_t = N_{djt}^{\alpha} K_{djt}^{-\alpha} (1 - \alpha).$$

## Log wages and aggregate labor demand

- ▶ Firm-level labor demand translates directly to city-level aggregate labor demand.
- ▶ Canceling capital out from the three equations in the previous slide, we have

$$w_{jt}^H = \ln W_{jt}^H = c_t + (1-\rho) \ln N_{jt} + (\rho-1) \ln H_{jt} + \ln(f_H(H_{jt}, L_{jt})) + \epsilon_{jt}^H,$$

$$w_{jt}^L = \ln W_{jt}^L = c_t + (1-\rho) \ln N_{jt} + (\rho-1) \ln L_{jt} + \ln(f_L(H_{jt}, L_{jt})) + \epsilon_{jt}^L,$$

$$N_{jt} = (\exp(\epsilon_{jt}^L) f_L(H_{jt}, L_{jt}) L_{jt}^\rho + \exp(\epsilon_{jt}^H) f_H(H_{jt}, L_{jt}) H_{jt}^\rho)^{\frac{1}{\rho}},$$

$$c_t = \ln \left( \alpha \left( \frac{1-\alpha}{\kappa_t} \right)^{\frac{1-\alpha}{\alpha}} \right).$$

## Eventually a reduced-form

- ▶ Instead of parameterizing  $f_H$  and  $f_L$ , let's rewrite wages as unknown functions  $g_H$  and  $g_L$  of  $H_{jt}$  and  $L_{jt}$

$$w_{jt}^H = g_H(H_{jt}, L_{jt}) + \epsilon_{jt}^H,$$

$$w_{jt}^L = g_L(H_{jt}, L_{jt}) + \epsilon_{jt}^L.$$

- ▶ Then Diamond "approximate" these functions using log-linear aggregate labor demand

$$w_{jt}^H = \gamma_{HH} \ln H_{jt} + \gamma_{HL} \ln L_{jt} + \epsilon_{jt}^H, \quad (1)$$

$$w_{jt}^L = \gamma_{LH} \ln H_{jt} + \gamma_{LL} \ln L_{jt} + \epsilon_{jt}^L. \quad (2)$$

- ▶ But, somehow the author shows an estimate for  $\rho$ .
  - ▶ We will see it later.

## Preferences

- ▶ A worker of skill level  $edu$  living in city  $j$  in year  $t$  inelastically supplies one unit of labor and earns a wage of  $W_{jt}^{edu}$ .
- ▶ The worker consumes
  - ▶ a local good  $M$ , which has a local price of  $R_{jt}$ ,
  - ▶ a national good  $O$ , which has a national price  $P_t$ ,and gains utility from the vector of amenities  $\mathbf{A}_{jt}$  in the city.
- ▶ The worker maximizes his Cobb-Douglas utility subject to his budget constraint

$$\begin{aligned} \max_{M,O} \quad & \ln(M^\zeta) + \ln(O^{1-\zeta}) + s_i(\mathbf{A}_{jt}) \\ \text{s.t.} \quad & P_t O + R_{jt} M \leq W_{jt}^{edu}. \end{aligned}$$

## Indirect utility and the local good demand

- ▶ Then the indirect utility is

$$\begin{aligned}V_{ijt} &= \ln \left( \frac{W_{jt}^{edu}}{P_t} \right) - \zeta \ln \left( \frac{R_{jt}}{P_t} \right) + s_i(\mathbf{A}_{jt}) \\ &= w_{jt}^{edu} - \zeta r_{jt} + s_i(\mathbf{A}_{jt}),\end{aligned}$$

where  $w_{jt} = \ln \left( \frac{W_{jt}^{edu}}{P_t} \right)$  and  $r_{jt} = \ln \left( \frac{R_{jt}}{P_t} \right)$ .

- ▶ The worker's local good demand is

$$HD_{ijt} = \frac{\zeta W_{jt}^{edu}}{R_{jt}}.$$

## Amenities (1)

- ▶ The function  $s_i(\mathbf{A}_{jt})$  maps the vector of city amenities  $\mathbf{A}_{jt}$  to the worker's utility value.
- ▶ It is specified as

$$s_i(A_{jt}) = a_{jt}\beta_i^a + x_{jt}^A\beta_i^x + \beta_i^{st}\mathbf{x}_j^{st} + \beta_i^{div}\mathbf{x}_j^{div} + \sigma_i\epsilon_{ijt}, \quad (3)$$

$$\beta_i^x = \beta^x \mathbf{z}_i,$$

$$\beta_i^a = \beta^a \mathbf{z}_i,$$

$$\beta_i^{st} = \mathbf{st}_i \beta^{st} \mathbf{z}_i,$$

$$\beta_i^{div} = \mathbf{div}_i \beta^{div} \mathbf{z}_i,$$

$$\sigma_i = \beta^\sigma \mathbf{z}_i,$$

$\epsilon_{ijt} \sim$  Type I Extreme Value.

## Amenities (2)

$$s_i(A_{jt}) = a_{jt}\beta_i^a + x_{jt}^A\beta_i^x + \beta_i^{st}\mathbf{x}_j^{st} + \beta_i^{\text{div}}\mathbf{x}_j^{\text{div}} + \sigma_i\epsilon_{ijt},$$

$$\beta_i^x = \beta^x\mathbf{z}_i,$$

$$\beta_i^a = \beta^a\mathbf{z}_i,$$

$$\beta_i^{st} = \mathbf{st}_i\beta^{st}\mathbf{z}_i,$$

$$\beta_i^{\text{div}} = \text{div}_i\beta^{\text{div}}\mathbf{z}_i,$$

$$\sigma_i = \beta^\sigma\mathbf{z}_i,$$

$\epsilon_{ijt} \sim$  Type I Extreme Value.

- ▶  $\beta_i^{st}$  and  $\beta_i^{\text{div}}$  measure worker  $i$ 's value of living in his state of birth and census division of birth, respectively.
- ▶  $\beta_i^x$  and  $\beta_i^a$  are the marginal utilities of the exogenous and endogenous amenities, respectively.
- ▶  $\mathbf{z}_i$  is a  $3 \times 1$  vector of dummy variable with each entry equal to 1 if the worker is white, black, or an immigrant, respectively.

## Amenities (3)

$$s_i(A_{jt}) = a_{jt}\beta_i^a + x_{jt}^A\beta_i^x + \beta_i^{st}\mathbf{x}_j^{st} + \beta_i^{div}\mathbf{x}_j^{div} + \sigma_i\epsilon_{ijt},$$

$$\beta_i^x = \beta^x \mathbf{z}_i,$$

$$\beta_i^a = \beta^a \mathbf{z}_i,$$

$$\beta_i^{st} = \mathbf{st}_i \beta^{st} \mathbf{z}_i,$$

$$\beta_i^{div} = \mathbf{div}_i \beta^{div} \mathbf{z}_i,$$

$$\sigma_i = \beta^\sigma \mathbf{z}_i,$$

$\epsilon_{ijt} \sim$  Type I Extreme Value.

- ▶ The coefficients  $(\beta^x, \beta^a, \beta^{st}, \beta^{div}, \beta^\sigma)$  are each  $1 \times 3$  vectors measuring the utility value of the city characteristic to the given demographic group.
- ▶  $\mathbf{x}_j^{st}$  is a  $1 \times 50$  binary vector where each element  $k$  is equal to 1 if part of city  $j$  is contained in state  $k$ .
- ▶  $\mathbf{x}_j^{div}$  is similarly defined for 9 census divisions.

## Amenities (3)

$$s_i(A_{jt}) = a_{jt}\beta_i^a + x_{jt}^A\beta_i^x + \beta_i^{st}\mathbf{x}_j^{st} + \beta_i^{div}\mathbf{x}_j^{div} + \sigma_i\epsilon_{ijt},$$

$$\beta_i^{st} = \mathbf{st}_i\beta^{st}\mathbf{z}_i,$$

$$\beta_i^{div} = \mathbf{div}_i\beta^{div}\mathbf{z}_i,$$

- ▶  $\mathbf{st}_i$  is a  $50 \times 1$  vector where each element is 1 if worker  $i$  was born in that state.
- ▶  $\mathbf{div}_i$  is defined similarly for census divisions.

## Amenities (4)

$$s_i(A_{jt}) = a_{jt}\beta_i^a + x_{jt}^A\beta_i^x + \beta_i^{st}\mathbf{x}_j^{st} + \beta_i^{\text{div}}\mathbf{x}_j^{\text{div}} + \sigma_i\epsilon_{ijt},$$

$$\sigma_i = \beta^\sigma \mathbf{z}_i,$$

$\epsilon_{ijt} \sim$  Type I Extreme Value.

- ▶ Each worker has an idiosyncratic taste for cities amenities  $\epsilon_{ijt}$ .
- ▶ The variance of workers' idiosyncratic tastes for each city differs across demographic groups.

## Normalization and indirect utility

- ▶ Renormalize the utility function by dividing each worker's utility by  $\beta^\sigma \mathbf{z}_i$ .
- ▶ With a slight abuse of notation, redefine the parameters of the re-normalized utility function using the same notation of the utility function measured in (log) wage units.
- ▶ Then the (renormalized) indirect utility is

$$V_{ijt} = (w_{jt}^{edu} - \zeta r_{jt}) \beta^w \mathbf{z}_i + a_{jt} \beta_i^a + \mathbf{x}_{jt}^A \beta_i^x + \beta_i^{st} \mathbf{x}_j^{st} + \beta_i^{div} \mathbf{x}_j^{div} + \epsilon_{ijt}.$$

## Rewriting indirect utility

- ▶ Define  $\delta_{jt}^z$  as utility value of the components of city  $j$  which all workers of type  $z$  identically

$$\delta_{jt}^z = (w_{jt}^{edu} + \zeta r_{jt})\beta^w z + a_{jt}\beta^a \mathbf{z} + x_{jt}^A \beta^x \mathbf{z}.$$

- ▶ Using this, the indirect utility is

$$V_{ijt} = \delta_{jt}^z + \mathbf{x}_j^{st} \beta^{st} \mathbf{z}_i + \mathbf{x}_j^{div} \text{div}_i \beta^{div} \mathbf{z}_i + \epsilon_{ijt}.$$

## Total expected populations

- ▶ The total high and low skill populations of city  $j$  are

$$H_{jt} = \sum_{i \in \mathcal{H}_t} \frac{\exp(\delta_{jt}^{z_i} + \mathbf{x}_j^{st} \mathbf{st}_i \beta^{st} \mathbf{z}_i + \mathbf{x}_j^{div} \mathbf{div}_i \beta^{div} \mathbf{z}_i)}{\sum_k^J \exp(\delta_{kt}^{z_i} + \mathbf{x}_k^{st} \mathbf{st}_i \beta^{st} \mathbf{z}_i + \mathbf{x}_k^{div} \mathbf{div}_i \beta^{div} \mathbf{z}_i)},$$

$$L_{jt} = \sum_{i \in \mathcal{L}_t} \frac{\exp(\delta_{jt}^{z_i} + \mathbf{x}_j^{st} \mathbf{st}_i \beta^{st} \mathbf{z}_i + \mathbf{x}_j^{div} \mathbf{div}_i \beta^{div} \mathbf{z}_i)}{\sum_k^J \exp(\delta_{kt}^{z_i} + \mathbf{x}_k^{st} \mathbf{st}_i \beta^{st} \mathbf{z}_i + \mathbf{x}_k^{div} \mathbf{div}_i \beta^{div} \mathbf{z}_i)}.$$

- ▶  $\mathcal{H}_t$  and  $\mathcal{L}_t$  are the set of high and low skill workers in the nation, respectively.

## Housing supply (1)

- ▶ Local prices,  $R_{jt}$ , are set through equilibrium in the housing market.
- ▶ The local housing price  $P_{jt}^{\text{house}}$  is

$$P_{jt}^{\text{house}} = MC(CC_{jt}, LC_{jt}),$$

where

- ▶  $CC_{jt}$  is local construction costs,
- ▶  $LC_{jt}$  is local land costs.
- ▶ Imagine the asset market steady state equilibrium. The rent is a fixed share of the housing price.
- ▶ Then local rents are

$$R_{jt} = \iota_t \times MC(CC_{jt}, LC_{jt}),$$

where  $\iota_t$  is the interest rate.

- ▶ Housing is owned by absentee landlords who rent the housing to local residents.

## Housing supply (2)

- ▶ The cost of land  $LC_{jt}$  is a function of the aggregate demand for local goods.
- ▶ The author parameterizes the log housing supply equation as

$$r_{jt} = \ln(R_{jt}) = \ln(\iota_t) + \ln(CC_{jt}) + \gamma_j \ln(HD_{jt}), \quad (4)$$

$$\gamma_j = \gamma + \gamma^{geo} \exp(x_j^{geo}) + \gamma^{reg} \exp(x_j^{reg}),$$

$$HD_{jt} = L_{jt} \frac{\zeta W_{jt}^L}{R_{jt}} + H_{jt} \frac{\zeta W_{jt}^H}{R_{jt}},$$

- ▶  $HD_{jt}$ : the aggregate local good demand in city  $j$  in year  $t$ ,
- ▶  $\gamma_j$ : the elasticity of rent with respect to local good demand,
- ▶  $x_j^{geo}$ : the share of land within 50km of each city's center which is unavailable for development,
- ▶  $x_j^{reg}$ : Saiz (2010)'s metropolitan area aggregates of the 2005 Wharton regulation index.

## Amenity supply

- ▶ Remember that roughly there are three kinds of amenities in (3)
  - ▶ exogenous amenities  $x_{jt}^A$ ,
  - ▶ endogenous amenities  $a_{jt}$ ,
  - ▶ state and census division fixed effects  $x_j^{st}$  and  $x_j^{\text{div}}$ .
- ▶ Now we specify  $a_{jt}$  as a function of college employment ratios

$$a_{jt} = \gamma^a \ln \left( \frac{H_{jt}}{L_{jt}} \right) + \epsilon_{jt}^a, \quad (5)$$

where

- ▶  $\gamma^a$  is the elasticity of amenity supply,
  - ▶  $\epsilon_{jt}^a$  is the exogenous component of the amenity index  $a_{jt}$ .
  - ▶ But, hasn't she already have exogenous amenities...?
- ▶ The vector of all amenities in the city,  $\mathbf{A}_{jt}$ , is

$$\mathbf{A}_{jt} = (\mathbf{x}_{jt}^A, \mathbf{x}_j^{st}, \mathbf{x}_j^{\text{div}}, a_{jt}).$$

## Equilibrium (1)

Equilibrium in this model is defined by a menu of wages rents, and high skill and low skill populations  $w_{jt}^L$ ,  $w_{jt}^H$ ,  $r_t$ ,  $H_{jt}$ ,  $L_{jt}$  such that

- ▶ The high skill labor demand equals high skill labor supply:

$$H_{jt} = \sum_{i \in \mathcal{H}_t} \frac{\exp(\delta_{jt}^{z_i} + \mathbf{x}_j^{st} \mathbf{st}_i \beta^{st} \mathbf{z}_i + \mathbf{x}_j^{div} \text{div}_i \beta^{div} \mathbf{z}_i)}{\sum_k^J \exp(\delta_{kt}^{z_i} + \mathbf{x}_k^{st} \mathbf{st}_i \beta^{st} \mathbf{z}_i + \mathbf{x}_k^{div} \text{div}_i \beta^{div} \mathbf{z}_i)},$$

$$w_{jt}^H = \gamma_{HH} \ln H_{jt} + \gamma_{HL} \ln L_{jt} + \epsilon_{jt}^H.$$

- ▶ The low skill labor demand equals low skill labor supply:

$$L_{jt} = \sum_{i \in \mathcal{L}_t} \frac{\exp(\delta_{jt}^{z_i} + \mathbf{x}_j^{st} \mathbf{st}_i \beta^{st} \mathbf{z}_i + \mathbf{x}_j^{div} \text{div}_i \beta^{div} \mathbf{z}_i)}{\sum_k^J \exp(\delta_{kt}^{z_i} + \mathbf{x}_k^{st} \mathbf{st}_i \beta^{st} \mathbf{z}_i + \mathbf{x}_k^{div} \text{div}_i \beta^{div} \mathbf{z}_i)},$$

$$w_{jt}^L = \gamma_{LH} \ln H_{jt} + \gamma_{LL} \ln L_{jt} + \epsilon_{jt}^L.$$

## Equilibrium (2)

- ▶ Housing demand equals housing supply:

$$r_{jt} = \ln(\iota_t) + \ln(CC_{jt}) + \gamma_j \ln(HD_{jt}),$$

$$\gamma_j = \gamma + \gamma^{geo} \exp(x_j^{geo}) + \gamma^{reg} \exp(x_j^{reg}),$$

$$HD_{jt} = L_{jt} \frac{\zeta \exp(w_{jt}^L)}{\exp(r_{jt})} + H_{jt} \frac{\zeta \exp(w_{jt}^H)}{\exp(r_{jt})}.$$

- ▶ Endogenous amenities demand equals endogenous amenity supply:

$$a_{jt} = \gamma^a \ln \left( \frac{H_{jt}}{L_{jt}} \right) + \epsilon_{jt}^a,$$

$$\delta_{jt}^z = \beta^w \mathbf{z}(w_{jt}^{edu} + \zeta r_{jt}) + \beta^x \mathbf{z}x_{jt}^A + \beta^a \mathbf{z}a_{jt}, \forall \mathbf{z}.$$

# Estimation

## Endogenous amenity index

- ▶ First the author constructs the endogenous amenity index  $a_{jt}$  and presents an instrumental variable which will be used later.
- ▶ The author collects 15 different amenities which are bucketed into 6 categories:
  - ▶ retail environment,
  - ▶ transportation infrastructure,
  - ▶ crime,
  - ▶ environmental quality,
  - ▶ school quality,
  - ▶ job quality (beyond wages).
- ▶ She uses principal component analysis (PCA) to combine the 15 data sources into a single index.
- ▶ She creates an amenity index using the first principal component within each amenity category.
- ▶ Then she creates an overall amenity index using the first principal component of all the amenity category indices.

## Principal component analysis (1)

- ▶ The author does not explicitly explain steps to compute amenity indices.
- ▶ I guess the following steps.
- ▶ Let  $N$  and  $K$  be the number of cities and the number of amenity characteristics, respectively.
- ▶ Let be  $X_i = (X_{i1}, \dots, X_{iK})$  be the characteristic vector of city  $i$ .
- ▶ Putting  $X_i$  for all cities  $i$ , we have a  $N \times K$  characteristic matrix  $X = (X_1, \dots, X_N)^T$ .
- ▶ We solve the following maximization problem to find a  $K$ -dimensional vector  $w$ :

$$\max_w w^T \Sigma w,$$

where

$$\Sigma = \frac{1}{N} X^T X,$$

$$\|w\| = 1.$$

## Principal component analysis (2)

- ▶ Then the amenity index  $a_i$  for city  $i$  is

$$a_i = w^T X_i.$$

- ▶ Actually the author does this in two steps: first within categories, and then overall.

TABLE 4—PRINCIPLE COMPONENT ANALYSIS FOR AMENITY INDICES

	Loading	Unexplained variance
<i>Panel A. Retail index</i>		
Apparel stores per 1,000 residents	0.653	0.411
Eating and drinking places per 1,000 residents	0.525	0.619
Movie theaters per 1,000 residents	0.545	0.591
<i>Panel B. Transportation index</i>		
Public buses per capita	0.566	0.5099
Public transit index	0.7015	0.2476
Average daily traffic—interstates	0.332	0.8315
Average daily traffic—major roads	0.277	0.8823
<i>Panel C. Crime index</i>		
Property crimes per 1,000 residents	0.707	0.395
Violent crimes per 1,000 residents	0.707	0.395
<i>Panel D. Environment index</i>		
Government spending on parks per capita	0.707	0.4541
EPA air quality index	-0.707	0.4541
<i>Panel E. School index</i>		
Government K-12 spending per student	0.707	0.3425
Student-teacher ratio	-0.707	0.3425
<i>Panel F. Job index</i>		
Patents per capita	0.707	0.4417
Employment rate	0.707	0.4417
<i>Panel G. Overall amenity index</i>		
Retail index	-0.2367	0.9039
Transportation index	0.4861	0.5948
Crime index	-0.1518	0.9605
Environment index	0.3973	0.7293
School index	0.5222	0.5323
Job index	0.5041	0.5643

*Notes:* All amenity data measured in logs. See online Appendix for detailed description of amenity data and their data sources. Panels A-F report weights used in each subindex construction. Panel G reports loadings on each subindex to create overall amenity index. See text for further details.

## Comments on the table

- ▶ Overall, the table has reasonable and interpretable results.
- ▶ But in Panel G, the retail index has a negative sign.
- ▶ The author acknowledges this issue and does not provide interpretation or rationale for it particularly.

## Bartik labor demand shocks (1)

- ▶ Cities' economic outcomes respond to plausibly exogenous shocks to local firms' productivities.
- ▶ The author uses changes in the productivity levels of the industries located within each city.
- ▶ Variation in productivity changes across industries differentially impact cities' productivities based on the industry composition.
- ▶ This is the idea of Bartik (1991). And this method was popularized by Autor et al. (2013).

## Bartik labor demand shocks (2)

- ▶ She constructs Bartik labor demand shocks as

$$\Delta B_{jt}^H = \sum_{ind} (w_{ind,-j,t}^H - w_{ind,-j,1980}^H) \frac{H_{ind,j,1980}}{H_{j,1980}},$$

$$\Delta B_{jt}^L = \sum_{ind} (w_{ind,-j,t}^L - w_{ind,-j,1980}^L) \frac{L_{ind,j,1980}}{L_{j,1980}},$$

where

- ▶  $w_{ind,-j,t}^H$  and  $w_{ind,-j,t}^L$  are the average log wage of high and low skill workers, respectively, in industry  $ind$  and year  $t$ , excluding workers in city  $j$  and workers within a city that has a border 25 miles of city  $j$ 's border,
- ▶  $H_{ind,j,1980}$  and  $L_{ind,j,1980}$  are the numbers of high and low skill workers in industry  $ind$  in year 1980.

## Inserting Bartik shocks into first differences

- ▶ Take first differences in (1) and (2).
  - ▶ Then we get first differences  $\Delta\epsilon_{jt}^H$  and  $\Delta\epsilon_{jt}^L$ .
- ▶ Assume that these Bartik labor demand shocks are a component of a city's exogenous productivity changes over time

$$\Delta\epsilon_{jt}^H = \gamma_{BHH}\Delta B_{jt}^H + \gamma_{BHL}\Delta B_{jt}^L + \Delta\tilde{\epsilon}_{jt}^H,$$

$$\Delta\epsilon_{jt}^L = \gamma_{BLH}\Delta B_{jt}^H + \gamma_{BLL}\Delta B_{jt}^L + \Delta\tilde{\epsilon}_{jt}^L.$$

- ▶ Then, overall, we have

$$\begin{aligned}\Delta w_{jt}^H &= \gamma_{HH}\Delta \ln H_{jt} + \gamma_{HL}\Delta \ln L_{jt} + \gamma_{BHH}\Delta B_{jt}^H + \gamma_{BHL}\Delta B_{jt}^L \\ &\quad + \Delta\tilde{\epsilon}_{jt}^H,\end{aligned}$$

$$\begin{aligned}\Delta w_{jt}^L &= \gamma_{LH}\Delta \ln H_{jt} + \gamma_{LL}\Delta \ln L_{jt} + \gamma_{BLH}\Delta B_{jt}^H + \gamma_{BLL}\Delta B_{jt}^L \\ &\quad + \Delta\tilde{\epsilon}_{jt}^L.\end{aligned}$$

## Exclusion restriction: housing market

- ▶ Conceptually, if the housing supply elasticity is high, an increase in local productivity leads to a big increase in employment and a small increase in (nominal) wages and rents.
- ▶ If the housing supply elasticity is low, an increase in local productivity leads to a small increase in employment and a big increase in (nominal) wages and rents.
- ▶ Guided by this intuition, the author proposes the following exclusion restriction

$$E(\Delta\tilde{\epsilon}_{jt}^H \Delta Z_{jt}) = 0,$$

$$E(\Delta\tilde{\epsilon}_{jt}^L \Delta Z_{jt}) = 0,$$

Instruments:

$$\Delta Z_{jt} \in \{\Delta B_{jt}^H x_j^{reg}, \Delta B_{jt}^L x_j^{reg}, \Delta B_{jt}^H x_j^{geo}, \Delta B_{jt}^L x_j^{geo}\}.$$

## Housing supply (1)

- ▶ Taking first differences in (4),

$$\begin{aligned}\Delta r_{jt} = & \Delta \ln(\iota_t) \\ & + (\gamma + \gamma^{geo} \exp(x_j^{geo}) + \gamma^{reg} \exp(x_j^{reg})) \Delta \ln(HD_{jt}) \\ & + \Delta \ln(CC_{jt}),\end{aligned}$$

$$HD_{jt} = L_{jt} \frac{\zeta W_{jt}^L}{R_{jt}} + H_{jt} \frac{\zeta W_{jt}^H}{R_{jt}}.$$

- ▶ Recall that  $\Delta \ln(CC_{jt})$  measures local changes in construction costs and other factors impacting housing prices not driven by population change.
- ▶  $\Delta \ln(CC_{jt})$  is not observed in the data.
  - ▶ The author treats this as an "error term."

## Housing supply (2)

- ▶ The moments restrictions are

$$E(\Delta \ln(CC_{jt})\Delta Z_{jt}) = 0,$$

Instruments:

$$\Delta Z_{jt} \in \left\{ \Delta B_{jt}^H, \Delta B_{jt}^L, \Delta B_{jt}^H x_j^{reg}, \Delta B_{jt}^L x_j^{reg}, \Delta B_{jt}^H x_j^{geo}, \Delta B_{jt}^L x_j^{geo} \right\}.$$

- ▶ I agree in that changes in construction costs  $\Delta \ln(CC_{jt})$  are orthogonal to changes in Bartik labor demands  $\Delta B_{jt}^H$  and  $\Delta B_{jt}^L$ .
- ▶ I don't know whether  $\Delta B_{jt}^H x_j^{reg}$  etc are nice instruments.
- ▶ On one hand, construction costs are obviously correlated to geographic features and regulation intensities.
- ▶ On the other hand, the author takes the product of the Bartik shocks and regulations/geographic features, for example,  $\Delta B_{jt}^H x_j^{reg}$ .

## Labor supply (1)

- ▶ Remember the indirect utility is specified as

$$V_{ijt} = \delta_{jt}^z + \beta^{st} \mathbf{z}_i \mathbf{st}_i \mathbf{x}_j^{st} + \beta^{\text{div}} \mathbf{z}_i \text{div}_i \mathbf{x}_j^{\text{div}} + \epsilon_{ijt},$$

$$\delta_{jt}^z = \beta^w \mathbf{z}_i (w_{jt}^{\text{edu}} - \zeta r_{jt}) + \beta^x \mathbf{z}_i \mathbf{x}_{jt}^A + \beta^a \mathbf{z}_i a_{jt}. \quad (6)$$

- ▶ The author follows the new BLP (Berry et al., 2004) and uses a two-step estimator.
- ▶ She uses a maximum likelihood estimator where she treats  $\delta_{jt}^z$  as a parameter.
  - ▶ She identifies the parameters in the first step with a "share" equation derived from the extreme value distribution.
- ▶ Then she breaks down  $\delta_{jt}^z$ .

## Labor supply (2)

- ▶ Take first differences in (6)

$$\Delta\delta_{jt}^z = \beta^w \mathbf{z}(\Delta w_{jt}^{edu} - \zeta \Delta r_{jt}) + \beta^x \mathbf{z} \Delta \mathbf{x}_{jt}^A + \beta^a \mathbf{z} \Delta a_{jt}.$$

- ▶ The author observes change in wages, rents and the amenity index.
- ▶ But, she says she doesn't observe the exogenous amenity changes.
  - ▶ This is because she pushed amenity features (education spending, crime etc) into  $a_{jt}$ , not  $\mathbf{x}_{jt}^A$ .
- ▶ Define  $\Delta\xi_{jt}^z$  as the change in utility value of city  $j$ 's amenities unobserved to the econometrician

$$\Delta\xi_{jt}^z = \beta^A \mathbf{z} \Delta \mathbf{x}_{jt}^A. \quad (7)$$

- ▶ Then we have

$$\Delta\delta_{jt}^z = \beta^w \mathbf{z}(\Delta w_{jt}^{edu} - \zeta \Delta r_{jt}) + \beta^a \mathbf{z} \Delta a_{jt} + \Delta\xi_{jt}^z.$$

- ▶ The author treats  $\Delta\xi_{jt}^z$  as an error term.

## Labor supply (3)

- ▶ The exclusion restriction is

$$E(\Delta\xi_{jt}^z \Delta Z_{jt}) = 0,$$

Instruments:

$\Delta Z_{jt} \in$

$$\{\Delta B_{jt}^H, \Delta B_{jt}^L, \Delta B_{jt}^H x_j^{reg}, \Delta B_{jt}^L x_j^{reg}, \Delta B_{jt}^H x_j^{geo}, \Delta B_{jt}^L x_j^{geo}, \Delta B_{jt}^L x_j^{geo}\}$$

- ▶ All instruments are orthogonal to changes in exogenous amenities  $\Delta\xi_{jt}^z$ .
- ▶ Changes in endogenous amenities  $\Delta a_{jt}$  will be specified later.
- ▶ Intuitively,  $\Delta B_{jt}^H$  and  $\Delta B_{jt}^L$  pushes up  $\Delta w_{jt}^{edu}$  through labor demand (not supply!).
- ▶ Intuitively,  $\Delta B_{jt}^{edu} x_j^{reg}$  and  $\Delta B_{jt}^{edu} x_j^{geo}$  pushes up  $\Delta r_{jt}$  through labor demand and housing market.

## Amenity supply

- ▶ Differencing the (endogenous) amenity supply equation (5) yields

$$\Delta a_{jt} = \gamma^a \Delta \ln \left( \frac{H_{jt}}{L_{jt}} \right) + \Delta \epsilon_{jt}^a.$$

- ▶ The exclusion restriction is

$$E(\Delta \epsilon_{jt}^a \Delta Z_{jt}) = 0,$$

Instruments:

$\Delta Z_{jt} \in$

$\{ \Delta B_{jt}^H, \Delta B_{jt}^L, \Delta B_{jt}^H x_j^{reg}, \Delta B_{jt}^L x_j^{reg}, \Delta B_{jt}^H x_j^{geo}, \Delta B_{jt}^L x_j^{geo}, \Delta B_{jt}^L x_j^{geo} \}$

- ▶ The author does not provide detailed intuition for these instruments for amenity shocks.
- ▶ My interpretation is that  $\Delta B_{jt}^H$ ,  $\Delta B_{jt}^H x_j^{reg}$ , and  $\Delta B_{jt}^H x_j^{geo}$  jointly affect  $\Delta H_{jt}$  through a "non-amenity" path.

# Eventually GMM

- ▶ All parameters are jointly estimated using two-step GMM.
  - ▶ Standard errors are clustered by MSA in all estimating equations.

## Estimation results

## Four models

1. The model without endogenous amenities (skill shares) with flexible  $\zeta$  (the standard model, column (1)),
  2. The model without endogenous amenities with fixed  $\zeta$  (the restricted standard model, column (2)),
  3. The model with endogenous amenities with fixed  $\zeta$  (the full model, column (3)),
  4. The model with endogenous amenities with flexible  $\zeta$  (she doesn't name this model, column (4)).
- ▶ For the models with fixed  $\zeta$ , the author uses the CEX data and sets  $\zeta = 0.62$ .
    - ▶ This number looks quite high.

TABLE 5—GMM ESTIMATES OF MODEL PARAMETERS

	Non-college	College	Non-college	College	Non-college	College	Non-college	College
	(1)		(2)		(3)		(4)	
<i>Panel A. Worker preferences for cities</i>								
Wage	4.155*** [0.603]	5.523*** [1.797]	3.757*** [0.561]	-1.783*** [0.682]	4.026*** [0.727]	2.116*** [1.146]	3.261*** [1.064]	4.976*** [1.671]
Rent	-2.418*** [0.349]	-1.404 [0.833]	-2.329*** [0.348]	1.105*** [0.423]	-2.496*** [0.451]	-1.312*** [0.711]	-2.944*** [0.551]	-2.159*** [0.821]
Implied local expenditure share	0.582*** [0.0678]	0.254** [0.078]	0.62 —	0.62 —	0.62 —	0.62 —	0.903*** [0.261]	0.434*** [0.0810]
Amenity index	—	—	—	—	0.274* [0.147]	1.012*** [0.115]	0.771*** [0.307]	0.638*** [0.185]
<i>Differential effects: Blacks</i>								
Wage	3.146*** [0.971]	7.852* [3.701]	0.299 [0.872]	2.549* [1.390]	1.681 [2.122]	5.423*** [2.019]	4.604*** [1.629]	8.882*** [4.059]
Rent	-0.620 [0.555]	-3.443* [1.637]	-0.173 [0.506]	-1.478* [0.806]	-0.975 [1.231]	-3.362*** [1.252]	0.181 [0.679]	-4.565*** [1.795]
Amenity index	—	—	—	—	0.741*** [0.221]	1.077*** [0.271]	-1.103*** [0.406]	0.551 [0.387]
<i>Differential effects: Immigrants</i>								
Wage	1.786 [1.157]	7.780** [3.259]	-3.872*** [1.066]	-4.022** [1.402]	0.307 [3.052]	0.942 [2.138]	1.682 [2.288]	7.054* [3.785]
Rent	1.324** [0.635]	-1.501 [1.361]	2.246** [0.618]	2.333 [0.813]	-0.190 [-1.893]	-0.594 [1.325]	1.490* [0.807]	-1.177 [1.510]
Amenity index	—	—	—	—	1.075*** [0.300]	0.982*** [0.238]	-0.544 [0.444]	-0.348 [0.358]

TABLE 5—GMM ESTIMATES OF MODEL PARAMETERS (*CONTINUED*)

	1	2	3	4
<i>Panel B. Housing Supply</i>				
Exp(Land use regulation)	0.084*** [0.020]	0.064*** [0.013]	0.091*** [0.019]	0.101*** [0.027]
Exp(Land unavailability)	0.019* [0.011]	0.014* [0.007]	0.021** [0.010]	0.025** [0.012]
Base house supply elasticity	0.002 [0.084]	0.063 [0.072]	0.014 [0.089]	-0.021 [0.102]
<i>Panel C. Labor demand</i>				
$\rho$	0.392*** [0.119]	0.393*** [0.1371]		
Elasticity of college wage w.r.t. college emp.			0.229 [0.307]	0.205 [0.320]
College wage w.r.t. noncollege emp.			0.312 [0.367]	0.376 [0.388]
Noncollege wage w.r.t. noncollege emp.			-0.552*** [0.202]	-0.448*** [0.196]
Noncollege wage w.r.t. college emp.			0.697*** [0.163]	0.642*** [0.172]
<i>Panel D. Amenity supply</i>				
College emp. ratio			2.60** [1.13]	2.65*** [1.107]
Hansen's J ( <i>p</i> -value):	0.0185	0.0095	0.135	0.213
$\chi^2$ test: estimates = calibrated local expenditure model estimates ( <i>p</i> -value):	0.0000			0.489
Endogenous amenity index			—	—
Calibrated local good expenditure share		—	—	—
CES labor demand	—	—		
Reduced-form labor demand			—	—

Counterfactuals (actually accounting exercises)

## Accounting exercises 1: productivity through wages (1)

- ▶ The author assess the contributions of productivity, amenities, and housing supply elasticities to the changes in cities' college employment ratios.
- ▶ First, she asks how much the productivity change alone affected the college employment ratios only through wages.
- ▶ To do so, she computes counterfactual college and noncollege wages in 2000 by

$$\hat{w}_{j,2000}^H = \underbrace{\gamma_{HH} \ln H_{j,1980} + \gamma_{HL} \ln L_{j,1980}}_{\text{Labor supply in 1980}} + \underbrace{\epsilon_{j,2000}^H}_{\text{Exog. productivity in 2000}},$$
$$\hat{w}_{j,2000}^L = \underbrace{\gamma_{LH} \ln H_{j,1980} + \gamma_{LL} \ln L_{j,1980}}_{\text{Labor supply in 1980}} + \underbrace{\epsilon_{j,2000}^L}_{\text{Exog. productivity in 2000}}.$$

## Accounting exercises 1: productivity through wages (2)

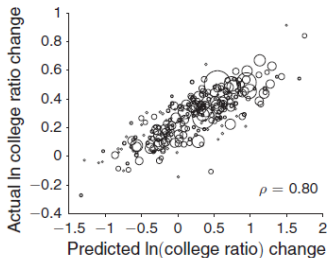
- ▶ Then she inserts such predicted wages into utility

$$V_{ijt} = \beta^w \mathbf{z}_i (\hat{w}_{j,2000}^{edu} - \zeta r_{j,1980}) + \beta^a \mathbf{z}_i a_{j,1980} + \xi_{j,1980}^z \\ + \beta^{st} \mathbf{z}_i \mathbf{st}_i \mathbf{x}_j^{st} + \beta^{div} \mathbf{z}_i \mathbf{div}_i \mathbf{x}_j^{div} + \epsilon_{i,j,1980}$$

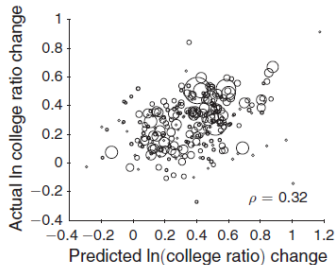
keeping other things fixed at the 1980 levels.

- ▶ Go back to (7) for the definition of  $\xi_{j,1980}^z$ . It's the city-year-demographic group fixed effect.
- ▶ Then given these utilities, she can compute college and noncollege populations across cities.

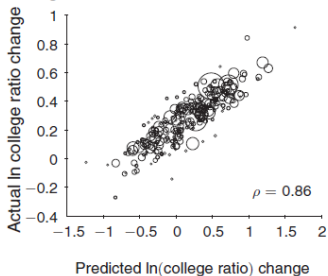
Panel A. Predicted change in ln college ratio due only to productivity changes



Panel B. Predicted change in ln college ratio due to observed wage and rent changes



Panel C. Predicted change in ln college ratio due to observed changes in wage, rent, and endogenous amenities



## Accounting exercises 2: factual wages and rents as of 2000

- ▶ The second counterfactual is inserting actual wages and rents as of 2000 and keeping all other variables at the 1980 levels

$$V_{ijt} = \beta^w \mathbf{z}_i (w_{j,2000}^{edu} - \zeta r_{j,2000}) + \xi_{j,1980}^z + \beta^a \mathbf{z}_i a_{j,1980} \\ + \beta^{st} \mathbf{z}_i \mathbf{st}_i \mathbf{x}_j^{st} + \beta^{div} \mathbf{z}_i \mathbf{div}_i \mathbf{x}_j^{div} + \epsilon_{i,j,1980}.$$

- ▶ And the author computes counterfactual equilibrium high and low skill populations across cities using these utilities.
- ▶ See Panel B.

## Accounting exercises 3: factual wages, rents, and endogenous amenities

- ▶ The third (and last) counterfactual is injecting the actual 2000 levels of wages, rents, and college employment ratios (as arguments of endogenous amenities) and keeping exogenous amenities at the 1980 levels

$$V_{ijt} = \beta^w \mathbf{z}_i (w_{j,2000}^{edu} - \zeta r_{j,2000}) + \xi_{j,1980}^z + \beta^a \mathbf{z}_i \hat{a}_{j,2000} \\ + \beta^{st} \mathbf{z}_i \mathbf{st}_i \mathbf{x}_j^{st} + \beta^{div} \mathbf{z}_i \mathbf{div}_i \mathbf{x}_j^{div} + \epsilon_{i,j,1980}, \\ \hat{a}_{j,2000} = \gamma^a \ln \left( \frac{H_{j,2000}}{L_{j,2000}} \right) + \epsilon_{j,1980}^a.$$

- ▶ And again the author computes high and low skill populations across cities associated with such utilities.
- ▶ See Panel C.

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