

# Final Exam

Motoaki Takahashi

February 5, 2026

## General instructions

- This is a closed-book exam. You may not see any textbook, note, PC, smartphone, or tablet.
- Write your answer clearly with a pencil or a pen.
- Write your name and student number in the first page of your answer sheets.
- The total points are 100, which accounts for 50% of the final grade.
- You must write your answer in English.

## 1 The Fréchet Distribution

(35 pts) There are  $N$  locations. An individual chooses one location to live in. Specifically, she solves

$$\max_{i \in \{1, \dots, N\}} v_i \epsilon_i,$$

where  $v_i > 0$  is a parameter of deterministic utility she can enjoy in location  $i$ , and  $\epsilon_i$  is a random variable governing idiosyncratic amenity shocks.

Assume that the idiosyncratic amenity shocks,  $\epsilon_i$ , follow the Fréchet distribution whose distribution function is

$$F(x) = e^{-x^{-\theta}}$$

independently across locations. Here  $\theta > 0$  is a shape parameter of this distribution.

Derive the probability that the individual chooses to live in location  $i$

$$P[v_i \epsilon_i > v_j \epsilon_j \text{ for any } j \neq i]$$

step by step.

## 2 Unit Dependence in the Hsieh-Moretti Model

(35 pts) Consider the Hsieh-Moretti model (Hsieh and Moretti, 2019). There are  $N$  cities. A mass  $L > 0$  of individuals exist in the entire economy. An equilibrium is a tuple of wages  $\{W_i\}_{i=1}^N$ , rents  $\{P_i\}_{i=1}^N$ , and population shares  $\{l_i\}_{i=1}^N$  ( $l_i$  denotes the population in city  $i$  relative to the total population  $L$ ) satisfying

$$\begin{aligned}W_i &= \mathcal{A}_i(l_i L)^{-\delta}, \\P_i &= \bar{P}_i(l_i L)^{\gamma_i}, \\l_i &= \frac{(W_i Z_i P_i^{-\beta})^\theta}{\sum_{j=1}^N (W_j Z_j P_j^{-\beta})^\theta},\end{aligned}$$

where  $\delta > 0$  is a parameter of the elasticity of nominal wages with respect to population,  $\mathcal{A}_i > 0$  is a parameter of productivity in location  $i$ ,  $\bar{P}_i$  is a parameter of a rent shifter in location  $i$ ,  $\gamma_i > 0$  is a parameter of the elasticity of rent with respect to population,  $\beta \in (0, 1)$  is a Cobb-Douglas parameter of the consumer expenditure share on housing, and  $\theta > 0$  is a shape parameter of Fréchet preference shocks. Most importantly,  $\gamma_i \neq \gamma_j$  for different cities  $i$  and  $j$ .

Do population shares  $\{l_i\}_{i=1}^N$  depend on units of the total population  $L$ ? Explain. (Hint: inflate or deflate the total population by a scalar  $a > 0$ .)

## 3 Small Questions

- (10 pts) In Diamond (2016), how does the author formulate endogenous amenities? Briefly explain.
- (10 pts) Matrix  $A$  is defined by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Write a Julia code to define such matrix  $A$ , and add 1 to the (1, 1) element, 2 to the (2, 2) element, and 3 to the (3, 3) element of matrix  $A$ .

- (10 pts) Greenstone et al. (2010) study the effect of a million-dollar plant on incumbent plants in the same county. To do so, what two groups of counties do the authors compare? Briefly explain.

## References

- Diamond, R. (2016). The determinants and welfare implications of us workers' diverging location choices by skill: 1980-2000. *American Economic Review*, 106(3):479–524.
- Greenstone, M., Hornbeck, R., and Moretti, E. (2010). Identifying agglomeration spillovers: Evidence from winners and losers of large plant openings. *Journal of Political Economy*, 118(3):536–598.
- Hsieh, C.-T. and Moretti, E. (2019). Housing constraints and spatial misallocation. *American Economic Journal: Macroeconomics*, 11(2):1–39.