Term Paper Questions

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General instructions

- Answer all questions.
- Submit one PDF file containing all your answers, one Julia file for Question 3 (Computing the Eaton-Kortum model), and one latex file if you create your term paper with latex.
- If you create a term paper with latex, you will get 5 extra points. In this case, you need to submit a tex file as well as a PDF file and a Julia file.
- See https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes for an introduction to latex.
- You can use Overleaf[1] to compile a latex file. Alternatively, you can use a software in your local computer to compile it.
- Send your term paper to `mtakahas@uni-mainz.de` by 10 pm on August 8 (Central European Time).
- You can submit your term paper only once. Resubmission is not allowed.

1 The Dornbusch-Fischer-Samuelson model

(10 pts) Consider the model described in "`1_DFS_slide.pdf`." We make one change. That is, now we assume that function $A(\cdot)$ is weakly decreasing, but not strictly decreasing (See p.3 of `1_DFS_slide.pdf`). Then for given $A(0) > \omega > A(1)$, is there only one $\tilde{z}$ such that $\omega = A(\tilde{z})$? Answer yes or no. If your answer is yes, give a proof. If your answer is no, give a counterexample.

Hint: Real-valued function f is weakly decreasing if $f(x) \geq f(x')$ for $x < x'$. f is strictly decreasing if $f(x) > f(x')$ for $x < x'$.

[1] https://www.overleaf.com/

2 Caliendo and Parro (2015)

Read Caliendo and Parro (2015) and answer the following questions.

1. (5 pts) What is the difference between the model in Caliendo and Parro (2015) and the one in Eaton and Kortum (2002)? Explain within 140 words.

2. (5 pts) How do Caliendo and Parro (2015) estimate the trade elasticity[2]? Explain with at least one equation.

3 Computing the Eaton-Kortum model

There are N countries. Consider an equilibrium of the Eaton-Kortum model characterized by the following $3N + N^2$ equations

$$w_i = \frac{1}{L_i} \sum_{n=1}^{N} w_n L_n \pi_{ni}, \tag{1}$$

$$\pi_{ni} = \frac{T_i (w_i^{\beta} p_i^{1-\beta} d_{ni})^{-\theta}}{\Phi_n}, \tag{2}$$

$$\Phi_n = \sum_{k=1}^{N} T_k (w_k^{\beta} p_k^{1-\beta} d_{nk})^{-\theta}, \tag{3}$$

$$p_n = \gamma \Phi_n^{-\frac{1}{\theta}}. \tag{4}$$

Notations follow the slides on the EK model (`2_EK_slide.pdf`). The outline of an algorithm to solve the equilibrium is as follows. Let ϵ_w and ϵ_p be small numbers such as $\epsilon_w > \epsilon_p > 0$.[3]

1. Guess $(w_i)_{i=1}^{N}$.
   (a) Guess $(p_i)_{i=1}^{N}$.
   (b) Given $(w_i)_{i=1}^{N}$ and $(p_i)_{i=1}^{N}$, compute $(\Phi_i)_{i=1}^{N}$ using (3).
   (c) Given $(\Phi_i)_{i=1}^{N}$, compute new price indices, say $(p_i^{\text{new}})_{i=1}^{N}$, using (4). If $\max_{i=1,\cdots,N} \left| \frac{p_i^{\text{new}} - p_i}{p_i} \right| \leq \epsilon_p$, go to the next step. Otherwise, that is, if $\max_{i=1,\cdots,N} \left| \frac{p_i^{\text{new}} - p_i}{p_i} \right| > \epsilon_p$, go back to (a), replacing $(p_i)_{i=1}^{N}$ with $(p_i^{\text{new}})_{i=1}^{N}$ as your new guess.

2. You have the converged $(p_i)_{i=1}^{N}$ given $(w_i)_{i=1}^{N}$. Compute $(\pi_{ni})_{n=1,i=1}^{N,N}$ using (2).

[2] In their notation, θ^j. j indexes industries.

[3] Practically, we use numbers such as 10^{-6} or 10^{-7}.

3. Given $(\pi_{ni})_{n=1,i=1}^{N,N}$, compute new nominal wages, say $(w_i^{\text{new}})_{i=1}^{N}$, using (1). If $\max_{i=1,\cdots,N} \left| \frac{w_i^{\text{new}} - w_i}{w_i} \right| \leq \epsilon_w$, stop. You got the converged $(w_i)_{i=1}^{N}$. Otherwise, that is, if $\max_{i=1,\cdots,N} \left| \frac{w_i^{\text{new}} - w_i}{w_i} \right| > \epsilon_w$, go back to 1., replacing $(w_i)_{i=1}^{N}$ with $(w_i^{\text{new}})_{i=1}^{N}$. Make sure that you set one numeraire, for example, set the first country's nominal wage to be one.

Consider the following setup. There are three countries, that is, $N = 3$. The populations are

$$L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}.$$

The labor share in value-added is $\beta = 0.5$. The trade elasticity is $\theta = 4$. The productivity, or strictly speaking, the location parameters of the productivity distributions are

$$T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ 1 \\ 1 \end{bmatrix}.$$

The trade costs are

$$d = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} = \begin{bmatrix} 1 & d_{12} & 2 \\ d_{21} & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$

Recall that d_{ni} is the trade costs for varieties from country i to country n (read the subscripts from the right to the left). $\gamma = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} = 0.65$ supposing $\sigma = 4$. Note that we have not assigned values to T_1, d_{12}, and d_{21}.

3.1 Comparative statics with respect to T_1

(20 pts) Assume that $d_{12} = d_{21} = 2$. Suppose that the productivity in country 1, T_1, varies from 0.5 to 0.51, 0.52, $\cdots$, 1.5 (that is, it varies from 0.5 to 1.5 with the step size 0.01). Plot the real wage $\frac{w_i}{p_i}$ $(i = 1, 2, 3)$ against T_1 for such a domain of T_1. That is, draw a graph where the horizontal axis is T_1 and the vertical axis is the real wages $\frac{w_i}{p_i}$ $(i = 1, 2, 3)$.

3.2 Comparative statics with respect to $d_{12} = d_{21}$

(20 pts) Assume that $T_1 = 1$. Assume that the trade costs between 1 and 2 are symmetric, and let $d^* = d_{12} = d_{21}$. Suppose that d^* varies from 1.5 to 1.51, 1.52, $\cdots$, 2.5 (that is, it varies from 1.5 to 2.5 with the step size 0.01). Plot the real wage $\frac{w_i}{p_i}$ $(i = 1, 2, 3)$ against d^* for such a domain of d^*. That is, draw a graph where the horizontal axis is d^* and the vertical axis is the real wages $\frac{w_i}{p_i}$ $(i = 1, 2, 3)$.

4 The Melitz-Chaney model

(10 pts) Consider the model described in `5_Melitz_Chaney.pdf`. Here, instead of the probability distribution in p.14 of `5_Melitz_Chaney.pdf`, we assume the following cumulative distribution function for the productivity distribution in country i

$$G_i(\varphi) = \begin{cases} 0 & \text{if } \varphi < \underline{\varphi}_i, \\ 1 - \left(\frac{\varphi}{\underline{\varphi}_i}\right)^{-\theta_i} & \text{if } \varphi \geq \underline{\varphi}_i, \end{cases} \tag{5}$$

where $\underline{\varphi}_i$ denotes an exogenous parameter of the lowest possible productivity that a firm in country i can draw. Assume that $\underline{\varphi}_i > 0$ and that for any i and j, $\underline{\varphi}_i < \varphi^*_{ij}$, that is, the lowest possible productivity in country i is lower than the exporting cutoff productivity from country i to country j. For the productivity distribution (5), derive the trade value from country i to country j, which should be a counterpart to equation (9) in p.17, `5_Melitz_Chaney.pdf`. Is it increasing or decreasing in $\underline{\varphi}_i$?

References

Caliendo, L. and Parro, F. (2015). Estimates of the trade and welfare effects of NAFTA. *The Review of Economic Studies*, 82(1):1–44.

Eaton, J. and Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5):1741–1779.