Term Paper Questions

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July 5, 2025

General instructions

- Answer all questions.
- Submit one PDF file containing all your answers, one Julia file for Question 2 (Computing the Eaton-Kortum model), and one latex file if you create your term paper with latex.
- If you create a term paper with latex, you will get 5 extra points. In this case, you need to submit a tex file as well as a PDF file and a Julia file.
- See https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes for an introduction to latex.
- You can use Overleaf[1] to compile a latex file. Alternatively, you can use a software in your local computer to compile it.
- Send your term paper to `mtakahas@uni-mainz.de` by 11 pm on July 24 (Central European Time).
- You can submit your term paper only once. Resubmission is not allowed.

1 The Dornbusch-Fischer-Samuelson model

(10 pts) Consider the model described in "`1_DFS_slide.pdf`." Remember that goods z are indexed by real numbers in $[0,1]$. Suppose that in the initial equilibrium, the relative unit labor requirement function (see p. 3 in the slides) is

$$A_1(z) = 2 - z$$

[1] https://www.overleaf.com/

for $z \in [0,1]$. Then in the second equilibrium, the relative unit labor requirement function is

$$A_2(z) = 3 - z.$$

In the third equilibrium, the relative unit labor requirement function is

$$A_3(z) = 2 - 1.5z.$$

Compare the initial equilibrium with the second and third equilibria by drawing a graph (or multiple graphs) like the figure in p. 10 in the slides. Economic variables that you need to discuss are the cutoff good ($\bar{z}$ in the figure) and the relative nominal wage ($\bar{\omega}$ in the figure). Briefly discuss economic intuitions on the differences among the three equilibria.

2 Computing the Eaton-Kortum model

There are N countries. Consider an equilibrium of the Eaton-Kortum model characterized by the following $3N + N^2$ equations

$$w_i = \frac{1}{L_i} \sum_{n=1}^{N} w_n L_n \pi_{ni}, \tag{1}$$

$$\pi_{ni} = \frac{T_i (w_i^{\beta} p_i^{1-\beta} d_{ni})^{-\theta}}{\Phi_n}, \tag{2}$$

$$\Phi_n = \sum_{k=1}^{N} T_k (w_k^{\beta} p_k^{1-\beta} d_{nk})^{-\theta}, \tag{3}$$

$$p_n = \gamma \Phi_n^{-\frac{1}{\theta}}. \tag{4}$$

Notations follow the slides on the EK model (`2_EK_slide.pdf`). The outline of an algorithm to solve the equilibrium is as follows. Let ϵ_w and ϵ_p be small numbers such as $\epsilon_w > \epsilon_p > 0$.[2]

1. Guess $(w_i)_{i=1}^N$.
   (a) Guess $(p_i)_{i=1}^N$.
   (b) Given $(w_i)_{i=1}^N$ and $(p_i)_{i=1}^N$, compute $(\Phi_i)_{i=1}^N$ using (3).
   (c) Given $(\Phi_i)_{i=1}^N$, compute new price indices, say $(p_i^{\text{new}})_{i=1}^N$, using (4). If $\max_{i=1,\cdots,N} \left| \frac{p_i^{\text{new}} - p_i}{p_i} \right| \leq \epsilon_p$, go to the next step. Otherwise, that is, if $\max_{i=1,\cdots,N} \left| \frac{p_i^{\text{new}} - p_i}{p_i} \right| > \epsilon_p$, go back to (a), replacing $(p_i)_{i=1}^N$ with $(p_i^{\text{new}})_{i=1}^N$ as your new guess.

[2] Practically, we use numbers such as 10^{-6} or 10^{-7}.

2. You have the converged $(p_i)_{i=1}^N$ given $(w_i)_{i=1}^N$. Compute $(\pi_{ni})_{n=1,i=1}^{N,N}$ using (2).

3. Given $(\pi_{ni})_{n=1,i=1}^{N,N}$, compute new nominal wages, say $(w_i^{\text{new}})_{i=1}^N$, using (1). If $\max_{i=1,\cdots,N} \left|\frac{w_i^{\text{new}}-w_i}{w_i}\right| \leq \epsilon_w$, stop. You got the converged $(w_i)_{i=1}^N$. Otherwise, that is, if $\max_{i=1,\cdots,N} \left|\frac{w_i^{\text{new}}-w_i}{w_i}\right| > \epsilon_w$, go back to 1., replacing $(w_i)_{i=1}^N$ with $(w_i^{\text{new}})_{i=1}^N$. Make sure that you set one numeraire, for example, set the first country's nominal wage to be one.

Consider the following setup. There are three countries, that is, $N = 3$. The populations are

$$L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}.$$

The labor share in value-added is $\beta = 0.5$. The trade elasticity is $\theta = 4$. The productivity, or strictly speaking, the location parameters of the productivity distributions are

$$T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ 2 \\ 2 \end{bmatrix}.$$

The trade costs are

$$d = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} = \begin{bmatrix} 1 & d_{12} & 2 \\ d_{21} & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$

Recall that d_{ni} is the trade costs for varieties from country i to country n (read the subscripts from the right to the left). $\gamma = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} = 0.65$ supposing $\sigma = 4$. Note that we have not assigned values to L, T_1, d_{12}, and d_{21}.

2.1 Comparative statics with respect to T_1

Assume that $d_{12} = d_{21} = 2$. Suppose that the productivity in country 1, T_1, varies from 1.5 to 1.51, 1.52, $\cdots$, 2.5 (that is, it varies from 1.5 to 2.5 with the step size 0.01). For each of the following population vectors L, plot real wages $\frac{w_i}{p_i}$ $(i = 1, 2, 3)$ against T_1 for such a domain of T_1. In your graphs, the horizontal axis must be T_1, and the vertical axis must be real wages $\frac{w_i}{p_i}$ $(i = 1, 2, 3)$.

1. (10 pts)

$$L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}.$$

2. (10 pts)

$$L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 100 \\ 80 \end{bmatrix}.$$

2.2 Comparative statics with respect to $d_{12} = d_{21}$

Assume that $T_1 = 2$. Assume that the trade costs between 1 and 2 are symmetric, and let $d^* = d_{12} = d_{21}$. Suppose that d^* varies from 1.5 to 1.51, 1.52, $\cdots$, 2.5 (that is, it varies from 1.5 to 2.5 with the step size 0.01). For each of the following population vectors L, plot real wages $\frac{w_i}{p_i}$ $(i = 1, 2, 3)$ against d^* for such a domain of d^*. In your graphs, the horizontal axis must be d^*, and the vertical axis must be real wages $\frac{w_i}{p_i}$ $(i = 1, 2, 3)$.

1. (6 pts)

$$L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}.$$

2. (7 pts)

$$L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 100 \\ 100 \end{bmatrix}.$$

3. (7 pts)

$$L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 100 \\ 80 \end{bmatrix}.$$

3 The Melitz-Chaney model

1. (4 pts) See p.8 of `5_Melitz_Chaney.pdf`. Solve the maximization problem

$$\max_{\{p_{ij}(\varphi)\}_{j \in S}} \sum_{j \in S} \left(p_{ij}(\varphi)^{1-\sigma} Y_j P_j^{\sigma-1} - \frac{w_i}{\varphi} \tau_{ij} p_{ij}(\varphi)^{-\sigma} Y_j P_j^{\sigma-1} - f_{ij} \right),$$

by taking the first-order conditions, and derive the optimal price

$$p_{ij}(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w_i}{\varphi} \tau_{ij}.$$

2. (6 pts) Derive the first equation in p.19 of `5_Melitz_Chaney.pdf`, that is,

$$f_i^e = \sum_{j \in S} \frac{\sigma - 1}{\theta_i + 1 - \sigma} \left(\frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{-\theta_i} \sigma^{-\frac{\theta_i}{\sigma-1}} f_{ij}^{\frac{\sigma-\theta_i-1}{\sigma-1}} Y_j^{\frac{\theta_i}{\sigma-1}} P_j^{\theta_i},$$

by computing the integration on the right-hand side of (10) in p.18.

4 Production networks

1. (5 pts) See p. 18 of `7_production_network.pdf`. Derive (6) by solving the cost minimization problem.

2. (5 pts) Suppose that there are 2 sectors. Assume that sector 2's (log) TFP shifter function is

$$a_2(\alpha_2) = -\sum_{i=1}^{2} (\alpha_{2i} - 0.4)^2 - (\alpha_{21} - \alpha_{22} - 0.1)^2.$$

Compute the inverse matrix of the Hessian of a_2 and discuss whether intermediate goods from sectors 1 and 2 are substitutes or complements for sector 2. You may refer to p. 33-35 of `7_production_network.pdf`.