# Term Paper: Answer Key

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## 1 The Dornbusch-Fischer-Samuelson model

No. Suppose that $\omega = 3$ and that the function $A$ is defined by

$$A(z) = \begin{cases} -9z + 6 & \text{for } 0 \leq z \leq 1/3, \\ 3 & \text{for } 1/3 \leq z \leq 2/3, \\ -9z + 9 & \text{for } 2/3 \leq z \leq 1. \end{cases}$$

$A$ is continuous and weakly decreasing. For any $z \in [1/3, 2/3]$, $\omega = A(z)$ holds.

## 2 Caliendo and Parro (2015)

1. Eaton and Kortum (2002) consider only one tradeable sector but Caliendo and Parro (2015) consider multiple tradeable and non-tradeable sectors. Moreover, Caliendo and Parro (2015) have input-output linkages across such multiple sectors.

2. Let $X_{ni}^j$ be the trade value of sector $j$ goods from country $i$ to country $n$. Let $\theta^j$ be the trade elasticity of sector $j$. $\tilde{\tau}_{ni}^j$ denotes the gross tariff[1] for sector $j$ goods from $i$ to $n$. Then the econometric specification that Caliendo and Parro (2015) adopt is

$$\log\left(\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{in}^j X_{hi}^j X_{nh}^j}\right) = -\theta^j \log\left(\frac{\tilde{\tau}_{ni}^j \tilde{\tau}_{ih}^j \tilde{\tau}_{hn}^j}{\tilde{\tau}_{in}^j \tilde{\tau}_{hi}^j \tilde{\tau}_{nh}^j}\right) + \tilde{\epsilon}^j,$$

where $\tilde{\epsilon}^j$ is the error term.

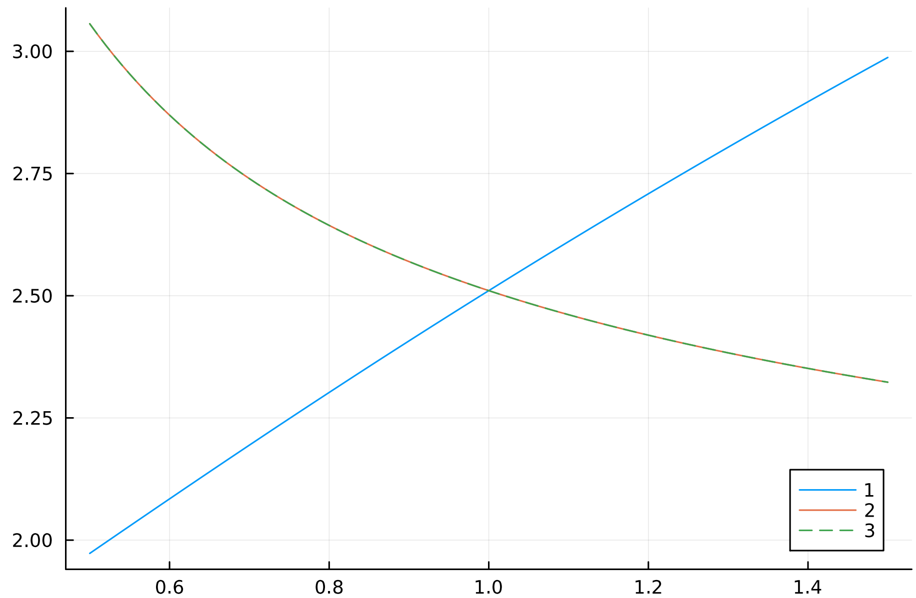
[1] If the tarrif rate is 30 percent, the gross tariff is 1.3.

# 3 Computing the Eaton-Kortum model

## 3.1 Comparative statics with respect to $T_1$

See Figure 1.

Figure 1: Comparative statics with respect to $T_1$




## 3.2 Comparative statics with respect to $d_{12} = d_{21}$

See Figure 2.

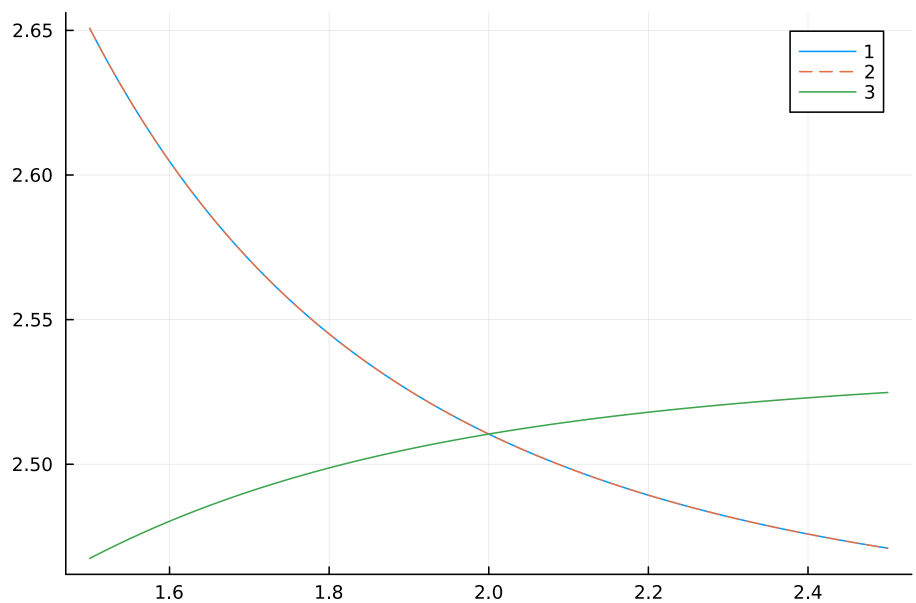
### A common mistake

Some students made a mistake in choosing a numeraire. In the outer loop for $w$, they choose country 1's labor for a numeraire. Therefore, country 1's wage is always one; $w_1 = 1$. But, at the same time, they set country 1's price index to be one; $p_1 = 1$. So they set country 1's composite of varieties to be a numeraire, too.

This is wrong. In the system of general equilibrium, only one good or production factor should be chosen as a numeraire. If you set country 1's labor to be a numeraire, other goods or production factors (including composites of varieties in country 1) should NOT be a numeraire.

Consider an economy with three kinds of good: apples, oranges, and pineapples. If you pick apples for a numeraire, oranges and pineapples should not be a numeraire. Choosing both labor and a composite of varieties in country 1 as numeraires is like choosing both apples and oranges as numeraires.

Figure 2: Comparative statics with respect to $d_{12} = d_{21}$



## 4 The Melitz-Chaney model

The cumulative distribution function of productivity in country $i$ is

$$G_i(\varphi) = 1 - \left(\frac{\varphi}{\underline{\varphi}_i}\right)^{-\theta_i}$$

for $\varphi \geq \underline{\varphi}_i$. Then the probability density function is

$$g_i(\varphi) = \theta_i \underline{\varphi}_i^{\theta_i} \varphi^{-\theta_i - 1}$$

for $\varphi \geq \underline{\varphi}_i$. The trade value from country $i$ to $j$ is

$$X_{ij} = \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} w_i^{1-\sigma} M_i \left(\int_{\varphi_{ij}^*} \varphi^{\sigma-1} dG_i(\varphi)\right) Y_j P_j^{\sigma-1}.$$

We need to compute $\int_{\varphi_{ij}^*} \varphi^{\sigma-1} dG_i(\varphi)$.

$$\begin{aligned}
\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} dG_i(\varphi) &= \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} g_i(\varphi) d\varphi \\
&= \theta_i \underline{\varphi}_i^{\theta_i} \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-\theta_i-2} d\varphi \\
&= \frac{\theta_i \underline{\varphi}_i^{\theta_i}}{\theta_i + 1 - \sigma} (\varphi_{ij}^*)^{\sigma-\theta_i-1} \\
&= \frac{\theta_i \underline{\varphi}_i^{\theta_i}}{\theta_i + 1 - \sigma} \left(\frac{\sigma f_{ij} (\frac{\sigma}{\sigma-1} w_i \tau_{ij})^{\sigma-1}}{Y_j P_j^{\sigma-1}}\right)^{\frac{\sigma-\theta_i-1}{\sigma-1}}.
\end{aligned}$$

Therefore, the trade value is

$$\begin{aligned}
X_{ij} &= \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \tau_{ij}^{1-\sigma} w_i^{1-\sigma} M_i \left(\frac{\theta_i \underline{\varphi}_i^{\theta_i}}{\theta_i + 1 + \sigma} \left(\frac{\sigma f_{ij} (\frac{\sigma}{\sigma-1} w_i \tau_{ij})^{\sigma-1}}{Y_j P_j^{\sigma-1}}\right)^{\frac{\sigma-\theta_i-1}{\sigma-1}}\right) Y_j P_j^{\sigma-1} \\
&= C_{1,i} \underline{\varphi}_i^{\theta_i} (\tau_{ij} w_i)^{-\theta_i} f_{ij}^{\frac{\sigma-\theta_i-1}{\sigma-1}} M_i (Y_j P_j^{\sigma-1})^{\frac{\theta_i}{\sigma-1}},
\end{aligned}$$

where $C_{1,i} = \sigma^{\frac{\sigma-\theta_i-1}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta_i} \left(\frac{\theta_i}{\theta_i+1-\sigma}\right)$. Since $\theta_i > 0$, the trade value $X_{ij}$ is increasing in $\underline{\varphi}_i$.

## References


Caliendo, L. and Parro, F. (2015). Estimates of the trade and welfare effects of NAFTA. *The Review of Economic Studies*, 82(1):1–44.

Eaton, J. and Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5):1741–1779.